Correction de l'interro 1

exo 1

1. $\Delta = 2^2 - 4 \times 3 \times (-1)$

$\Delta = 4 + 12$

$\Delta = 16 > 0$

$x_1 = \frac{-2 - \sqrt{16}}{2 \times 3}$ $\quad x_2 = -\frac{2+4}{6}$

$x_1 = \frac{-2-4}{6}$ $\quad x_2 = \frac{1}{3}$

$x_1 = -1$

Donc $P(x) = 3(x+1)(x - \frac{1}{3})$

2. $5x^2 + 14x - 3 = 0$

$\Delta = 14^2 - 4 \times 5 \times (-3)$

$\Delta = 196 + 60$

$\Delta = 256 > 0$

$x_1 = \frac{-14 - \sqrt{256}}{2 \times 5}$

$x_1 = \frac{-14 - 16}{10}$ $\quad x_2 = -\frac{14+16}{10}$

$\boxed{x_1 = -3}$ $\quad \boxed{x_2 = \frac{1}{5}}$

exo 2

1. $P(2) = 2 \times 2^3 - 2 \times 2^2 - 16 \times 2 + 24$

$P(2) = 16 - 8 - 32 + 24$

$P(2) = 0$

Donc 2 est une racine évidente.

2. $P(x) = (x-2)(ax^2 + bx + c)$

$P(x) = ax^3 + bx^2 + cx - 2ax^2 - 2bx - 2c$

$P(x) = ax^3 + (b - 2a)x^2 + (c - 2b)x - 2c$

Par identification on a :

$$\begin{cases} a = 2 \\ b - 2a = -2 \\ c - 2b = -16 \\ -2c = 24 \end{cases} \Leftrightarrow \begin{cases} a = 2 \\ b = 2 \\ c = -12 \end{cases}$$

3. $P(x) = 0$

$(x-2)(2x^2 + 2x - 12) = 0$

Soit $x - 2 = 0$ $\quad$ Soit $2x^2 + 2x - 12 = 0$

$x = 2$ $\quad \Delta = 2^2 - 4 \times 2 \times (-12)$

$\Delta = 100$

$x_1 = \frac{-2 - 10}{4}$ $\quad x_2 = -\frac{2+10}{4}$

$x_1 = -3$ $\quad x_2 = 2$

Donc $S = \{-3; 2\}$

exo3

1. $(\sqrt{2}+\sqrt{7})^2 = \sqrt{2}^2 + 2\sqrt{2}\times\sqrt{7} + \sqrt{7}^2$

$= 2 + 2\sqrt{14} + 7$

$= 9 + 2\sqrt{14}$.

2. $\Delta = (\sqrt{2}-\sqrt{7})^2 - 4\times 1\times(-\sqrt{14})$

$\Delta = 9 - 2\sqrt{14} + 4\sqrt{14}$

$\Delta = 9 + 2\sqrt{14}$

$\Delta = (\sqrt{2}+\sqrt{7})^2 > 0$

$x_1 = \dfrac{-(\sqrt{2}-\sqrt{7}) - (\sqrt{2}+\sqrt{7})}{2}$

$x_1 = \dfrac{-\sqrt{2}+\sqrt{7}-\sqrt{2}-\sqrt{7}}{2}$

$\boxed{x_1 = -\sqrt{2}}$

$x_2 = \dfrac{-(\sqrt{2}-\sqrt{7}) + (\sqrt{2}+\sqrt{7})}{2}$

$x_2 = \dfrac{-\sqrt{2}+\sqrt{7}+\sqrt{2}+\sqrt{7}}{2}$

$\boxed{x_2 = \sqrt{7}}$

exo4:

1a $P(3) = 0$

$\Leftrightarrow 3^2 + (m+4)\times 3 + 7m - 21 = 0$

$\Leftrightarrow 9 + 3m + 12 + 7m - 21 = 0$

$\Leftrightarrow 10m = 0$

$\Leftrightarrow m = 0$.

b. Pour $m=0$ $\quad P(x) = x^2 + 4x - 21$

$\Delta = 4^2 - 4\times 1\times(-21)$

$\Delta = 100 > 0$

$x_1 = \dfrac{-4-\sqrt{100}}{2}$

$x_1 = \dfrac{-4-10}{2}$ $\qquad x_2 = \dfrac{-4+10}{2}$

$\boxed{x_1 = -7}$ $\qquad \boxed{x_2 = 3}$ Donc -7 est l'autre solution

2a. $\Delta = (m+4)^2 - 4\times 1(7m-21)$

$\Delta = m^2 + 8m + 16 - 28m + 84$.

$\Delta = m^2 - 20m + 100$

$\Delta = (m-10)^2$

Posons $\Delta = 0$ donc $(m-10)^2 = 0 \Leftrightarrow \boxed{m = 10}$

b. Pour $m = 10$.

$$P(x) = x^2 + 14x + 49$$

$P(x) = 0 \iff x^2 + 14x + 49 = 0$ avec $\Delta = 0$

Donc la sol. est $x_0 = \frac{-14}{2 \times 1} = \boxed{-7}$.

3a. $P(-7) = (-7)^2 + (m+4)(-7) + 7m - 21$

$= 49 - 7m - 28 + 7m - 21$

$= 0$

Donc -7 est une racine de P $\forall m$.

b. $P(3-m) = (3-m)^2 + (m+4)(3-m) + 7m - 21$

$= 9 - 6m + m^2 + 3m - m^2 + 12 - 4m + 7m - 21$

$= 0$.

c. Donc $P(x) = (x - x_1)(x - x_2)$

$= (x+7)(x - 3 + m)$