

## Correction de l'exercice 3

exo 1

1 a.  $u_{10} = u_3 + (10-3)r$       b.  $u_8 = u_3 + (8-3)r$   
 $53 = 18 + 7r$                        $u_8 = 18 + 5 \times 5$   
 $35 = 7r$                                $u_8 = 43$   
 $r = 5$

2 a.  $v_6 = v_3 \times q^{6-3}$                $v_3 = v_0 \cdot q^3$   
 $-192 = -24 \times q^3$                    $-24 = v_0 \times 8$   
 $q^3 = 8$                                    $v_0 = -3$   
 $q = 2$

b.  $q = 2 > 1$  donc  $(2^n)$  est croissante.  
 $v_0 < 0$  donc  $(v_n)$  est décroissante.

exo 2

1.  $u_1 = \frac{2u_0}{2+3u_0}$                $u_2 = \frac{2u_1}{2+3u_1}$   
 $u_1 = \frac{2 \times 1}{2+3 \times 1}$                $u_2 = \frac{2 \times \frac{2}{5}}{2+3 \times \frac{2}{5}}$   
 $u_1 = \frac{2}{5}$                                $u_2 = \frac{1}{4}$

$u_1 - u_0 = \frac{2}{5} - 1 = -\frac{3}{5}$   
 $u_2 - u_1 = \frac{1}{4} - \frac{2}{5} = -\frac{3}{20}$

$\left. \begin{array}{l} u_1 - u_0 \neq u_2 - u_1 \\ \text{Donc } (u_n) \text{ non arithmétique} \end{array} \right\}$

$\left. \begin{array}{l} \frac{u_1}{u_0} = \frac{2}{5} \\ \frac{u_2}{u_1} = \frac{1/4}{2/5} = \frac{5}{8} \end{array} \right\} \begin{array}{l} \frac{u_1}{u_0} \neq \frac{u_2}{u_1} \\ \text{Donc } (u_n) \text{ non géométrique.} \end{array}$

2 a.  $v_{n+1} - v_n = \frac{1}{u_{n+1}} - \frac{1}{u_n}$   
 $= \frac{1}{\frac{2u_n}{2+3u_n}} - \frac{1}{u_n}$   
 $= \frac{2+3u_n}{2u_n} - \frac{2}{2u_n}$   
 $= \frac{3u_n}{2u_n} = \frac{3}{2}$

Donc  $(u_n)$  arithmétique de raison  $\frac{3}{2}$ .

b.  $v_0 = \frac{1}{u_0} = \frac{1}{1} = 1$   
 $v_n = v_0 + nr$   
 $v_n = 1 + \frac{3}{2}n$

c.  $v_n = \frac{1}{u_n}$  donc  $u_n = \frac{1}{v_n}$   
 $u_n = \frac{1}{1 + \frac{3}{2}n} = \frac{2}{2+3n}$

3.  $u_{n+1} - u_n = \frac{2}{2+3(n+1)} - \frac{2}{2+3n}$   
 $= \frac{2}{2+3n+3} - \frac{2}{2+3n}$   
 $= \frac{2}{5+3n} - \frac{2}{2+3n}$   
 $= \frac{2(2+3n) - 2(5+3n)}{(5+3n)(2+3n)}$   
 $= \frac{4+6n-10-6n}{(5+3n)(2+3n)}$

$$u_{n+1} - u_n = \frac{-6}{(5+3n)(2+3n)}$$

$-6 < 0$   
 $5+3n > 0$  et  $2+3n > 0$   
Donc  $u_{n+1} - u_n < 0$   
 $(u_n)$  est décroissante

$$4 \quad u_n = \frac{1}{25}$$

$$\frac{2}{2+3n} = \frac{1}{25}$$

$$2 \times 25 = 2 + 3n$$

$$50 = 2 + 3n$$

$$48 = 3n$$

$$n = \frac{48}{3}$$

$$n = 16$$

exo 3

$$1. \quad u_1 = \frac{3u_0 + 4}{u_0 + 3}$$

$$= \frac{3 \times 1 + 4}{1 + 3}$$

$$= \frac{7}{4}$$

$$u_2 = \frac{3u_1 + 4}{u_1 + 3}$$

$$= \frac{3 \times \frac{7}{4} + 4}{\frac{7}{4} + 3}$$

$$= \frac{\frac{21}{4} + 4}{\frac{7}{4} + 3}$$

$$= \frac{\frac{21 + 16}{4}}{\frac{7 + 12}{4}}$$

$$= \frac{37}{19}$$

$$u_3 = \frac{3u_2 + 4}{u_2 + 3}$$

$$= \frac{3 \times \frac{37}{19} + 4}{\frac{37}{19} + 3}$$

$$= \frac{\frac{111}{19} + 4}{\frac{37}{19} + 3}$$

$$= \frac{\frac{111 + 76}{19}}{\frac{37 + 57}{19}}$$

$$= \frac{187}{94}$$

$$2. \quad u_1 - u_0 = \frac{7}{4} - 1$$

$$= \frac{3}{4}$$

$$u_2 - u_1 = \frac{37}{19} - \frac{7}{4}$$

$$= \frac{15}{76}$$

$$\left. \begin{array}{l} u_1 - u_0 \neq u_2 - u_1 \\ \text{Dnc } (u_n) \text{ non} \\ \text{arithmétique} \end{array} \right\} \begin{array}{l} \frac{u_1}{u_0} = \frac{7}{4} \\ \frac{u_2}{u_1} = \frac{\frac{37}{19}}{\frac{7}{4}} \\ \frac{u_2}{u_1} = \frac{148}{133} \end{array} \left\{ \begin{array}{l} \frac{u_1}{u_0} \neq \frac{u_2}{u_1} \\ \text{Dnc } (u_n) \\ \text{non géométrique} \end{array} \right.$$

$$3. \quad v_{n+1} = \frac{u_{n+1} + 2}{u_{n+1} - 2}$$

$$= \frac{\frac{3u_n + 4}{u_n + 3} + 2}{\frac{3u_n + 4}{u_n + 3} - 2}$$

$$v_{n+1} = \frac{\frac{3u_n + 4}{u_n + 3} + \frac{2u_n + 6}{u_n + 3}}{\frac{3u_n + 4}{u_n + 3} - \frac{2u_n + 6}{u_n + 3}}$$

$$v_{n+1} = \frac{5u_n + 10}{3u_n + 4 - 2u_n - 6}$$

$$v_{n+1} = \frac{5u_n + 10}{u_n - 2} \times \frac{u_n + 3}{u_n + 3}$$

$$v_{n+1} = \frac{5(u_n + 2)}{u_n - 2}$$

$$v_{n+1} = 5 v_n$$

Dnc  $(v_n)$  geo  
de raison  $q=5$

$$4. \quad v_n = v_0 \cdot q^n \quad v_0 = \frac{u_0 + 2}{u_0 - 2} = \frac{1 + 2}{1 - 2} = -3$$

$$\text{Dnc } v_n = -3 \times 5^n$$

$$5. \quad v_n = \frac{u_{n+1} + 2}{u_{n+1} - 2}$$

$$v_n (u_{n+1} - 2) = u_{n+1} + 2$$

$$v_n \cdot u_{n+1} - 2v_n = u_{n+1} + 2$$

$$v_n \cdot u_n - u_n = 2 + 2v_n$$

$$u_n (v_n - 1) = 2 + 2v_n$$

$$u_n = \frac{2 + 2v_n}{v_n - 1}$$

$$u_n = \frac{2 + 2 \times (-3) \times 5^n}{-3 \times 5^n - 1}$$

$$u_n = \frac{2 - 6 \times 5^n \times (-1)}{-3 \times 5^n - 1 \times (-1)}$$

$$u_n = \frac{6 \times 5^n - 2}{1 + 3 \times 5^n}$$

$$6. \quad u_6 = \frac{6 \times 5^6 - 2}{1 + 3 \times 5^6}$$

$$u_6 = \frac{23437}{11719}$$

$$u_6 \approx 1,9999 \text{ à } 10^{-4} \text{ près}$$