Correction du DST 3

exo 1

1C 2D 3B 4D

explications :

1) $u_{12} = u_7 + 5r$

$29 = 14 + 5r$

$5r = 15$

$r = 3$

$u_{12} = u_{10} + 2r$

$29 = u_{10} + 2\times 3$

$u_{10} = 29 - 6$

$u_{10} = 23$

2) $-15\% \to \times\left(1 - \frac{15}{100}\right) = \times 0,85$ Donc $u_{n+1} = 0,85 u_n$

3) $S = 101 \, \frac{200 + 300}{2}$

$= 15150$

4. $u_2 = 2u_1 - 4$

$5 = 2u_1 - 4$

$u_1 = 6$

$u_1 = 2u_0 - 4$

$6 = 2u_0 - 4$

$u_0 = 5$

exo 3

$u_{n+1} - u_n = \frac{2(n+1)}{n+1-1} - \frac{2n}{n-1}$

$= \frac{2n+2}{n} - \frac{2n}{n-1}$

$= \frac{(2n+2)(n-1) - 2n\times n}{n(n-1)}$

$= \frac{2n^2 - 2n + 2n - 2 - 2n^2}{n(n-1)}$

$= \frac{-2}{n(n-1)}$

or $n \geq 2$ Donc $n > 0$

et $n - 1 > 0$

$-2 < 0$

Donc $(u_n) \searrow$ car

$u_{n+1} - u_n < 0$

exo 2

1. $u_1 = u_0 - \frac{1}{0+1} = 1 - 1 = 0$

$u_2 = u_1 - \frac{1}{1+1} = 0 - \frac{1}{2} = -\frac{1}{2}$

$u_3 = u_2 - \frac{1}{2+1} = -\frac{1}{2} - \frac{1}{3} = -\frac{5}{6}$.

2. $u_{n+1} - u_n = -\frac{1}{n+1} < 0$ car $n \geq 0$ Donc $(u_n) \searrow$

exo 5

1. a. $u_1 = \left(1 - \frac{20}{100}\right) u_0 + 500$

$= 0,8 \times 5000 + 500$

$= 4500$.

b. En 2024, on calcule $u_2 = 0,8 u_1 + 500$

$= 0,8 \times 4500 + 500$

$= 4100$

2. $\left.\begin{array}{l} u_1 - u_0 = -500 \\ u_2 - u_1 = -400 \end{array}\right\}$ (u_n) n'est pas arithmétique.

$\left.\begin{array}{l} \frac{u_1}{u_0} = 0,9 \\ \frac{u_2}{u_1} \approx 0,91 \end{array}\right\}$ (u_n) n'est pas géométrique.

3. Perte de 20% → $\left(1-\frac{20}{100}\right) \times u_n = 0,8\,u_n$.

Gain de 500 → $0,8\,u_n + 500$.

Donc $u_{n+1} = 0,8\,u_n + 500$

4.a $v_{n+1} = u_{n+1} - 2500$

$= 0,8\,u_n + 500 - 2500$

$= 0,8\,u_n - 2000$

$= 0,8\,(u_n - 2500)$

$= 0,8\,v_n$

Donc (v_n) SG de raison $q = 0,8$.

Son 1er terme est $v_0 = u_0 - 2500$

$= 5000 - 2500$

$= 2500$.

b. $v_n = v_0 \cdot q^n$

$v_n = 2500 \times 0,8^n$

c. $v_n = u_n - 2500$

$u_n = v_n + 2500$

$u_n = 2500 \times 0,8^n + 2500$

4. En 2035, $n = 13$ $u_{13} = 2500 \times 0,8^{13} + 2500$

$= 2637$ abonnés

exo 4

1) $u_{n+2} = \dfrac{u_{n+1}+1}{u_{n+1}-1}$

$= \dfrac{\frac{u_n+1}{u_n-1}+1}{\frac{u_n+1}{u_n-1}-1}$

$= \dfrac{\frac{u_n+1+u_n-1}{u_n-1}}{\frac{u_n+1-u_n+1}{u_n-1}}$

$= \dfrac{2u_n}{u_n-1} \times \dfrac{u_n-1}{2}$

$= u_n$

2) Si n est pair

$u_n = u_0 = \boxed{2}$

Si n est impair

$u_n = u_1$.

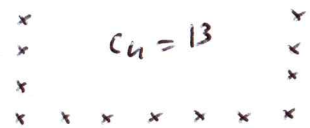
$u_1 = \dfrac{u_0+1}{u_0-1} = \dfrac{2+1}{2-1} = \boxed{3}$

exo 6

1. $C_1 = 1$

$C_2 = 5$

$C_3 = 9$

2.  $C_4 = 13$

3. Il semble que (C_n) soit arithmétique de raison 4

$C_{n+1} = C_n + 4$.

4. $C_n = C_1 + (n-1)r$

$C_n = 1 + 4(n-1)$

$C_n = 4n - 3$

5. Posons $4n - 3 = 6789$

$4n = 6792$

$n = 1698$.

Donc oui et le rang de la figure est le 1698e.