Correction DST 2

exo 1

1.

x	-3	-1	3	8
$f(x)$	1	2	-2	1
$f'(x)$	2	0	-1	4

2. T_{-3} : $y = f'(-3)(x+3) + f(-3)$

$y = 2(x+3)+1$

$y = 2x+7$

T_{-1} : $y = f'(-1)(x+1) + f(-1)$

$y = 0(x+1)+2$

$y = 2$

exo 2

1 a $\tau = \dfrac{f(1+h) - f(1)}{h}$

$\tau = \dfrac{3(1+h)^2 - (1+h) - (3\times 1^2 - 1)}{h}$

$\tau = \dfrac{3(1+2h+h^2) - 1 - h - 3 + 1}{h}$

$\tau = \dfrac{3+6h+3h^2-1-h-2}{h}$

$\tau = \dfrac{3h^2+5h}{h}$

$\tau = 3h+5$

$\lim\limits_{h\to 0} (3h+5) = 5$

Donc $f'(1) = 5$

b. $\tau = \dfrac{f(5+h) - f(5)}{h}$

$\tau = \dfrac{\sqrt{2(5+h)-1} - \sqrt{2\times 5 - 1}}{h}$

$\tau = \dfrac{\sqrt{9+2h} - \sqrt{9}}{h}$

$\tau = \dfrac{(\sqrt{9+2h} - \sqrt{9})(\sqrt{9+2h} + \sqrt{9})}{h(\sqrt{9+2h} + \sqrt{9})}$

$\tau = \dfrac{\sqrt{9+2h}^2 - \sqrt{9}^2}{h(\sqrt{9+2h} + \sqrt{9})}$

$\tau = \dfrac{9+2h-9}{h(\sqrt{9+2h} + \sqrt{9})}$

$\tau = \dfrac{2}{\sqrt{9+2h} + \sqrt{9}}$

$\lim\limits_{h\to 0} \dfrac{2}{\sqrt{9+2h}+\sqrt{9}} = \dfrac{2}{2\sqrt{9}} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3}$

Donc $f'(5) = \dfrac{1}{3}$.

2. $\tau = \dfrac{f(a+h) - f(a)}{h}$

$= \dfrac{\frac{1}{a+h} - \frac{1}{a}}{h}$

$= \dfrac{\frac{a}{(a+h)a} - \frac{a+h}{a(a+h)}}{h}$

$= \dfrac{\frac{a-a-h}{a(a+h)}}{h}$

$= \dfrac{-h}{a(a+h)} \times \dfrac{1}{h} = \dfrac{-1}{a(a+h)}$

$\lim\limits_{h\to 0} \dfrac{-1}{a(a+h)} = \dfrac{-1}{a^2}$

Donc $f'(a) = \dfrac{-1}{a^2}$

no 3

1. $g'(1) = 2$

2. On sait que $y = g'(1)(x-1) + g(1)$

$y = 2(x-1) + g(1)$

$y = 2x \underbrace{-2 + g(1)}_{5}$

Posons $-2 + g(1) = 5$

$g(1) = 7$.

no 4

1. $u_{n+1} - u_n = \frac{n+1-3}{2(n+1)+1} - \frac{n-3}{2n+1}$

$= \frac{n-2}{2n+3} - \frac{n-3}{2n+1}$

$= \frac{(n-2)(2n+1) - (n-3)(2n+3)}{(2n+3)(2n+1)}$

$= \frac{2n^2 + n - 4n - 2 - (2n^2 + 3n - 6n - 9)}{(2n+3)(2n+1)}$

$= \frac{2n^2 - 3n - 2 - 2n^2 + 3n + 9}{(2n+3)(2n+1)}$

$= \frac{7}{(2n+3)(2n+1)}$

2. Comme $n \in \mathbb{N}$, alors $2n+3 \geqslant 0$ et $2n+1 > 0$

$7 > 0$ Donc $u_{n+1} - u_n > 0$ $(u_n) \nearrow$

no 5.

1 a. $u_0 = 150$

$u_1 = 150 \times \left(1 - \frac{20}{100}\right) + 35$

$= 150 \times 0,8 + 35$

$= 155$

b. Perte de 20% $\rightarrow$ $u_n \times \left(1 - \frac{20}{100}\right) = 0,8\,u_n$

Gain de 35 vélos $\rightarrow 0,8u_n + 35$

Donc $u_{n+1} = 0,8u_n + 35$

2.a. $v_0 = u_0 - 175$

$= 150 - 175$

$= -25$

$t_0 = -25 \times 0,8^0$

$= -25 \times 1$

$= -25$

Donc $v_0 = t_0$

b. $v_{n+1} = u_{n+1} - 175$

$= 0,8u_n + 35 - 175$

$= 0,8u_n - 140$

$= 0,8\left(u_n - \frac{140}{0,8}\right)$

$= 0,8(u_n - 175)$

$= 0,8\,v_n$

$t_{n+1} = -25 \times 0,8^{n+1}$

$= -25 \times 0,8^n \times 0,8$

$= t_n \times 0,8$

$= 0,8\,t_n$

c Pour tout $n \in \mathbb{N}$, on a $v_n = t_n$.

3. $v_n = u_n - 175$

Donc $u_n = v_n + 175$

$u_n = t_n + 175$

$u_n = -25 \times 0,8^n + 175$

4. En 2029, $n = 11$

$u_{11} = -25 \times 0,8^{11} + 175$

$u_{11} \approx 173$ vélos